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LETTER TO THE EDITOR

Cosmological *f–g* fields relevant to quark confinement

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Abstract. It is shown that cosmological *f–g* fields are related to cosmological gravitational fields. By use of this relation, a method is presented for generating solutions of the *f–g* field equations.

The quark confinement mechanism of Salam and Strathdee (1977a) within the framework of the strong-gravity theory (Isham *et al* 1971) is well understood when the solutions of empty *f–g* field equations are also solutions of the Einstein empty field equations with a cosmological constant. In connection with this problem Salam and Strathdee (1977b) and Isham and Storey (1978) have recently made some attempts to find spherically symmetric solutions of coupled field equations. Isham and Storey (1978) have observed that, in the case of spherical symmetry, if one of the fields satisfies the empty Einstein field equations with a cosmological term, then the other field satisfies the same kind of equations as well. In this Letter we would like to report that the Isham–Storey observation is valid in general.

Given fields f, g, we can always write (Gürses 1979)

$$f^{\mu\nu} = \phi g^{\mu\nu} + H^{\mu\nu}, \tag{1}$$

where ϕ is a scalar function and $H^{\mu\nu}$ is a second-rank symmetric tensor. We note that $f^{\mu\nu}$ is invariant under the following transformations:

$$H^{\mu\nu} \to \tilde{H}^{\mu\nu} = H^{\mu\nu} - \psi g^{\mu\nu}, \tag{2}$$

$$\phi \to \tilde{\phi} = \phi + \psi, \tag{3}$$

where ψ is an arbitrary scalar function. We shall later use this freedom for simplification. Lowering the indices of $H^{\mu\nu}$ by $g_{\mu\nu}$ and using the notation of Isham and Storey we write down the *f–g* empty field equations‡:

$$G^f_{\mu\nu} = R^f_{\mu\nu} - \tfrac{1}{2}R^f f_{\mu\nu} = k_f^2 T^f_{\mu\nu}, \tag{4}$$

where

$$T^f_{\mu\nu} = \alpha^2 \Delta^u [vIf_{\mu\nu} + 2(3\phi - 3 + H)g_{\mu\nu} - 2H_{\mu\nu}] \tag{5}$$

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‡ The covariant second-rank tensor $f_{\mu\nu}$ which is the inverse of $f^{\mu\nu}$ should be distinguished from the object $g_{\mu\alpha}g_{\nu\beta}f^{\alpha\beta}$.

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and

$$G^g_{\mu\nu} = R^g_{\mu\nu} - \tfrac{1}{2}R^g g_{\mu\nu} = k_g{}^2 T^g_{\mu\nu}, \tag{6}$$

where

$$T^g_{\mu\nu} = \alpha^2\Delta^{-v}\{[-2\phi(3\phi-3+H)+uI]g_{\mu\nu} - 2(2\phi-3+H)H_{\mu\nu} + 2H^{\beta}{}_{\mu}H_{\beta\nu}\}, \tag{7}$$

with

$$I = -4(\phi-1)(3\phi-3+H) - H(2\phi-2+H) + H^{\alpha\beta}H_{\alpha\beta}, \tag{8}$$

$$\Delta = (\det g_{\mu\nu})(\det f_{\mu\nu})^{-1} \tag{9}$$

$$H = g_{\mu\nu}H^{\mu\nu}, \qquad \alpha = M/2k_f, \tag{10}$$

and u, v are constants appearing in the field equations with the constraint $u+v=\frac{1}{2}$, which ensures that the mixing term in the Lagrangian density is a tensor density of the correct weight (Isham and Storey 1978), M being a constant.

We shall now, without proof, state a theorem concerning the above empty-field equations.

Theorem. If $f^{\mu\nu}$ and $g^{\mu\nu}$ satisfy the empty f–g field equations (4) and (6) and if ϕ and $H^{\mu\nu}$ obey equation (1), then the following three statements are equivalent:

(a) $G^f_{\mu\nu} = \lambda_f f_{\mu\nu}$,

(b) $G^g_{\mu\nu} = \lambda_g g_{\mu\nu}$,

(c) $H^{\mu\alpha}H_{\nu\alpha} = aH^{\mu}{}_{\nu} + b\delta^{\mu}{}_{\nu}$, $\qquad a = 2\phi - 3 + H$,

where λ_f and λ_g are cosmological constants corresponding to the fields f and g, respectively, and a and b are scalars. We may choose the arbitrary function ψ in (2) and (3) so that the scalar b appearing in (c) can be taken as zero.

The proof of the above theorem does not depend on this reduction (fixation of the function ψ), but it simplifies the calculations considerably. With this simplification, we find that

$$H = na, \tag{11}$$

where n takes any integer value between one and three. The cosmological constants λ_f and λ_g are functions of ϕ and u (or v); hence the conformal factor ϕ is constant whenever one of the fields satisfies Einstein's vacuum field equation with cosmological constant. The algebraic relations among the components of $H^{\mu\nu}$ given in (c) enable us to find the inverse of $f^{\mu\nu}$. It reads†

$$f_{\mu\nu} = \phi^{-1}g_{\mu\nu} - \phi^{-1}(\phi+a)^{-1}H_{\mu\nu}. \tag{12}$$

General covariance of the f–g field theory requires simultaneous transformations of both fields under the general coordinate transformations (locking together property (Isham and Storey 1978)). Geometrically both fields may be isometric but the 'locking together property' prevents them from being identical. Utilising this fact and the above theorem we propose a method to generate a solution of the empty f–g field equations from a solution of the Einstein empty field equations with cosmological constant. The

† See previous footnote.

first step is to choose a solution $\tilde{g}_{\mu\nu}$ of Einstein's empty field equations with cosmological constant λ. The second step is to perform a regular coordinate transformation. If the transformation matrix is $S^{\mu}{}_{\nu} = \partial x^{\mu}/\partial x'^{\nu}$, then the metric in the new coordinate system (x'^{μ}) is given by

$$\tilde{g}'_{\mu\nu}(x') = S^{\alpha}{}_{\mu} S^{\beta}{}_{\nu} \tilde{g}_{\alpha\beta}(x). \tag{13}$$

The third step is to choose the new coordinates in such a way that (13) takes the form

$$\tilde{g}'_{\mu\nu}(x') = \phi^{-1} \bar{g}_{\mu\nu}(x') - \phi^{-1}(\phi + a)^{-1} H_{\mu\nu}, \tag{14}$$

where ϕ, a and $H_{\mu\nu}$ obey the conditions given in (c) and (11) and where $\bar{g}_{\mu\nu}(x')$ is also a solution of Einstein's vacuum field equations with a cosmological constant. The final step is to identify $\tilde{g}'_{\mu\nu}(x')$ in (14) as the f field and $\bar{g}_{\mu\nu}(x')$ as the g field. Hence $\lambda_f = \lambda$, where λ is the cosmological constant corresponding to the solution we started with.

We may divide the solutions obtainable by this method in two classes, according to whether the fields f and g are isometric or anisometric. The solutions found by Salam and Strathdee (1977b) and Isham and Storey (1978) fall into the first class. Each class may in turn be divided into two subclasses according to whether the scalar function a appearing in (c) vanishes or not. When $a = 0$, $H_{\mu\nu}$ becomes null, that is, $H_{\mu\nu} = l_\mu l_\nu$, where l_μ is a null vector.

Finally we would like to note that there exist at least three f fields corresponding to a g field. These f fields are obtainable by using three different coordinate transformations (14) which satisfy the constraint (11).

The detailed version of this work with further studies of the same subject will be published elsewhere.

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